

**Sample Paper – 2014**  
**Class – XII**  
**Subject – Mathematics**

M.M. 100

Time:3hrs.

**Instructions:** Question Number 1 to 10 carry 1 marks each, Question Number 11 to 22 carry 4 marks each, Question Number 23 to 29 carry 6 mark each.

**Section A**

1. Check the operation \* On Z, define by  $a * b = a^b$  for commutative.
2. Find the value of the expression:  $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ .
3. If  $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ , then find x.
4. If A is an invertible matrix of order 2, and  $\det(A)=5$ , then find  $\det(A^{-1})$ .
5. Compute the product :  $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$ .
- 6.
7. Find  $\lambda$  and  $\mu$  if  $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$ .
8. Find the Cartesian equation of the line which passes through the point (- 2, 4, -5) and parallel to the line given by  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-6}{2}$ .

9. Find the distance of each of the point (2, 3, -5) from the plane  $x + 2y - 2z = 9$ .

10. Evaluate  $\int \frac{x^2}{1-x^6} dx$ .

**Section B**

11. Consider the binary operation  $\vee$  on the set  $\{1, 2, 3, 4, 5\}$  defined by  $a \vee b = \min \{a, b\}$ . Write the operation table of the operation  $\vee$ . OR Show that the relation R defined in the set A of all polygons as  $R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same number of sides}\}$ , is an equivalence relation.

12. Prove that:  $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$ .

13. By using properties of determinants, show that 
$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

14. Find the value of a and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \geq 10 \end{cases}$$

is a continuous function.

15. If  $y = 3\cos(\log x) + 4\sin(\log x)$ , show that  $x^2 y_2 + x y_1 + y = 0$ .

16. Find the intervals in which the function  $f(x) = (x+1)^3(x-3)^3$  is strictly increasing or decreasing:

OR

Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:  $f(x) = \sin x - \cos x, 0 < x < 2\pi$

17. Evaluate:  $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$ .

18. Solve the Differential equation:  $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ .
19. Form the differential equation representing the family of curves given by:  $(x - a^2) + 2y^2 = a^2$ , where a is an arbitrary constant. OR Find a particular solution to the D.E.:  $\frac{dy}{dx} - 3y \cot x = \sin 2x$ ;  $y = 2$  when  $x = \frac{\pi}{2}$ .
20. Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$  and  $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , and  $\vec{c} \cdot \vec{d} = 15$ .
21. Find the distance of the point (-1, -5, -10) from the point of intersection of the line  $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .
22. How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

OR

Two groups are competing for the position on the Board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4, respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.

### Section C

23. Solve system of linear equation by using matrix method :  $2x + y + z = 1$ ,  $x - 2y - z = \frac{3}{2}$ ,  $3y - 5z = 9$
24. Prove that the volume of the largest cone that can be inscribed in sphere of radius R is  $\frac{8}{27}$  of the volume of the sphere.

OR

Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is  $\tan^{-1}\sqrt{2}$ .

25. Evaluate:  $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx.$

26. Using integration find the area of the triangular region whose sides have the equations:

$$y = 2x + 1, y = 3x + 1 \text{ and } x = 4.$$

27. Find the equation of the plane through the line of intersection of the planes  $x + y + z = 1$  and  $2x + 3y + 4z = 5$  which is perpendicular to the plane  $x - y + z = 0$ .

28. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

29. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs. Also find the mean and variance of the distribution.

OR

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black ball; One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

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